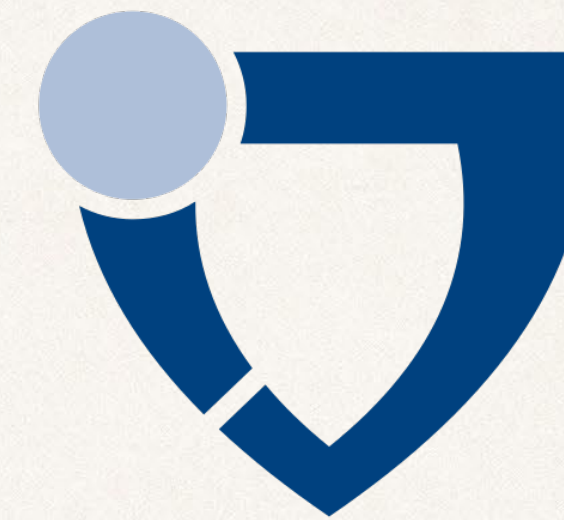


SHORTENING A TEST FOR SKILL ASSESSMENT

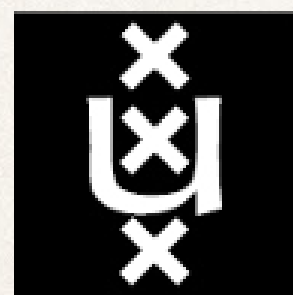


Students: Anna Panzeri, Cheyenne Cavender, Laura Van Hove, Teresa Gehrig, Christopher Schaar

Supervisor: Prof Pasquale Anselmi



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DI PADOVA

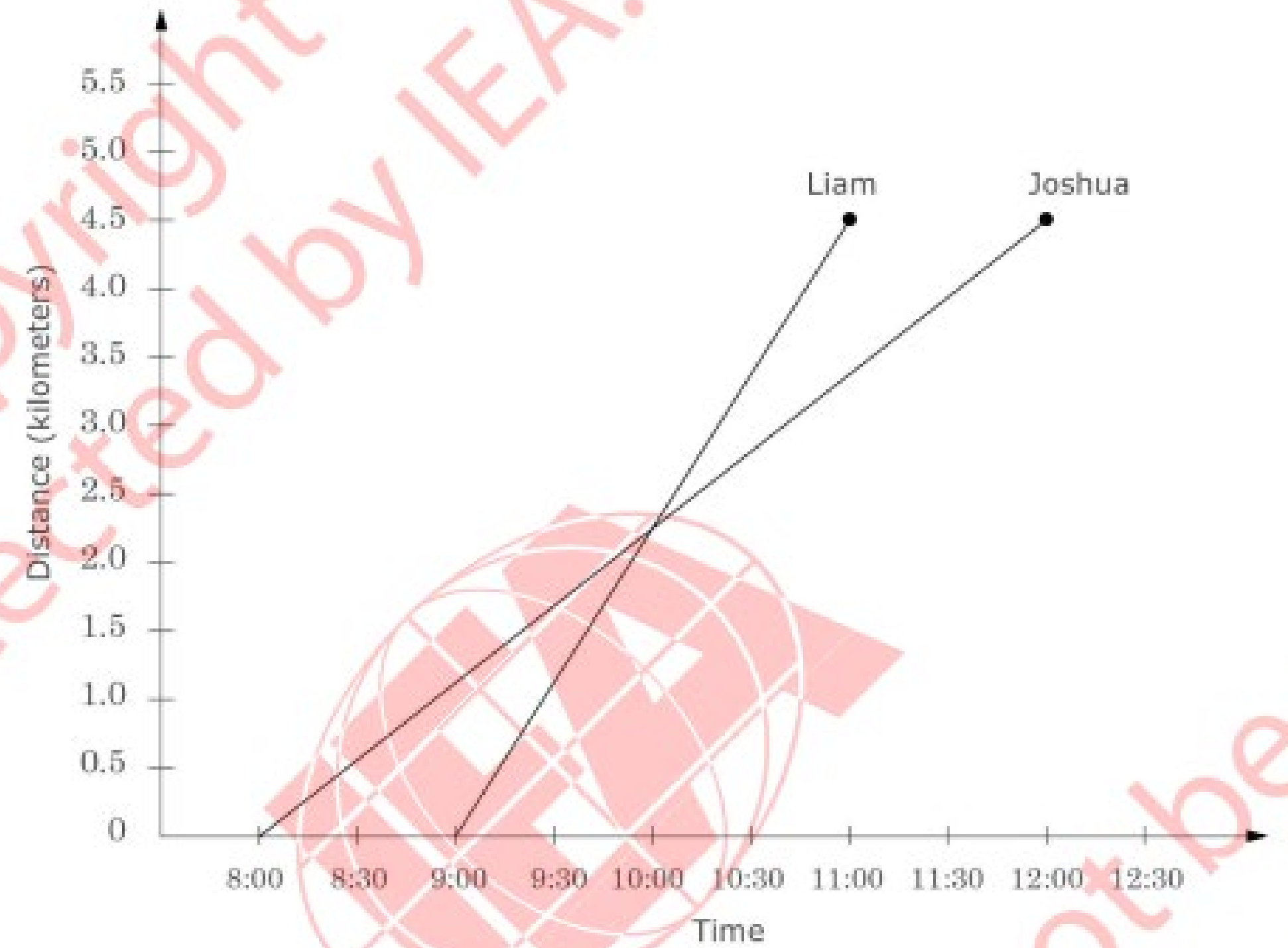
TIMSS, 2003

Trends In International Mathematics And Science Study (TIMSS), is an international assessment tool, that measures students' math, science and reading performance

(<https://timssandpirls.bc.edu/>)

Excerpt -->

The graph represents the distance and time of a hike taken by Joshua and Liam.



If they both started from the same place and walked in the same direction, at what time did they meet?

- (A) 8:00
- (B) 8:30
- (C) 9:00
- (D) 10:00
- (E) 11:00

1.
Create a skill map:
29 items * 14 skills

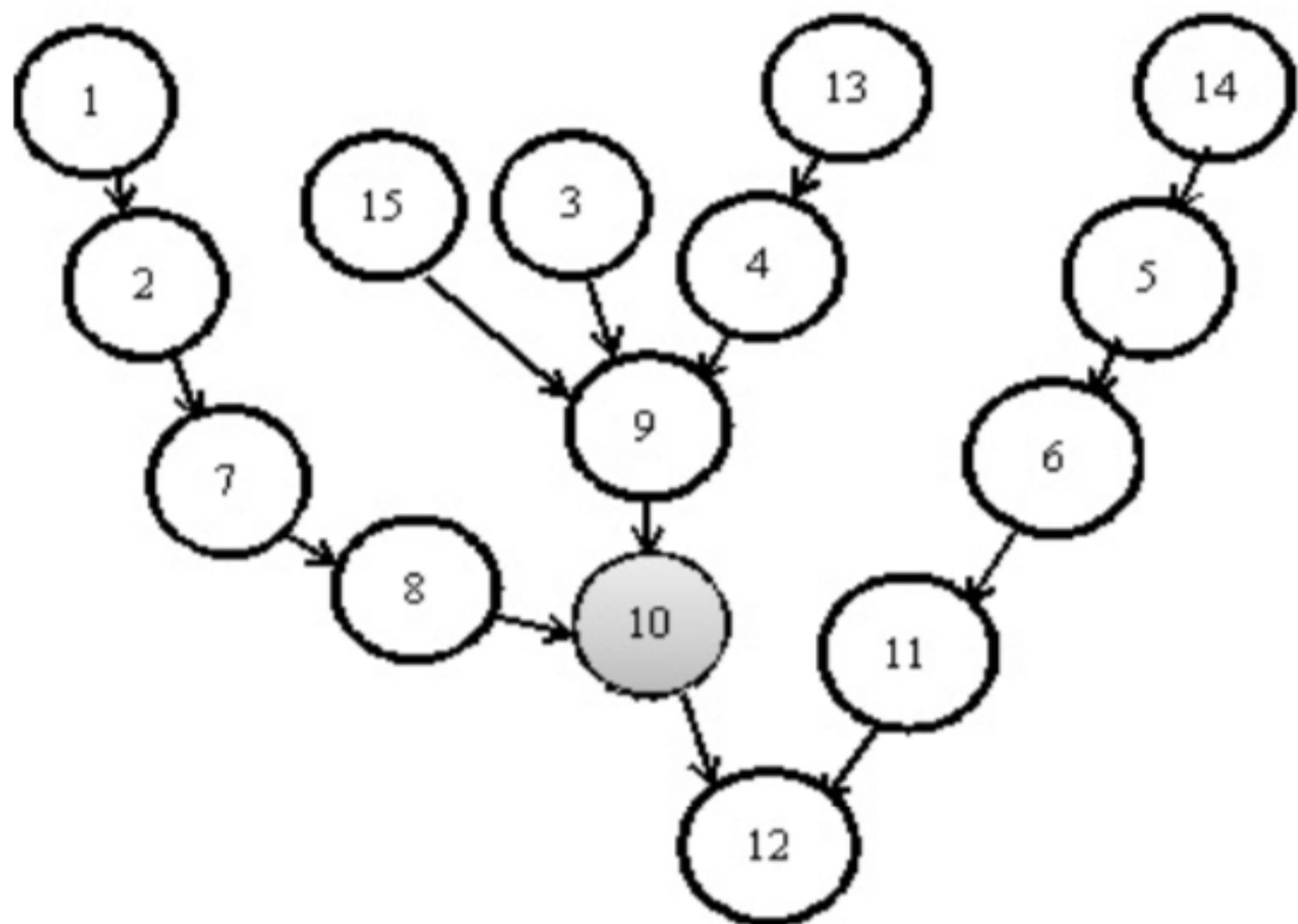


Figure 6: Hierarchical relationship among the attributes for booklet 1

```

map <- matrix( c(|
1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1,
2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
3, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
4, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0,
5, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
6, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0,
7, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
8, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
9, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
10, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0,
11, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
12, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
13, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
14, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
15, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
16, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
17, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1,
18, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
19, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
20, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
21, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
22, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
23, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
24, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
25, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
26, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
27, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0,
28, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0,
29, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0) ,
      29, 15, byrow = TRUE)
map
  
```

The Procedure (1)

1.
Create a skill map



2.
Create the base of the knowledge Space:
by looking at the graph with the 14 skills
The Base Consists of Atoms

```
# 1 2 3 4 5 6 7 8 9 11 12 13 14 15
base <- matrix (c(
  1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
  1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0,
  1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
  0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
  0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1,
  0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
  0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
  0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
  0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0,
  0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0,
  0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 0),
ncol=14, byrow= TRUE)
```

The Procedure (2)

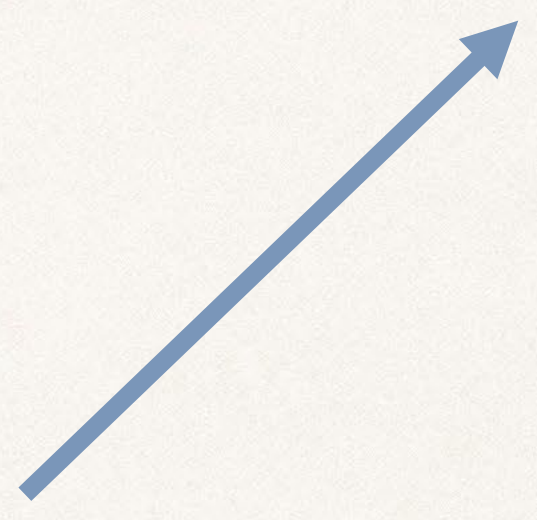
CLOSURE UNDER UNION: QHELP EXAMPLE

Not closed under union	Closed under union
R	R
R, JASP	R, JASP
Psychophysics	Psychophysics
-	R, Psychophysics
-	R, Psychophysics, JASP

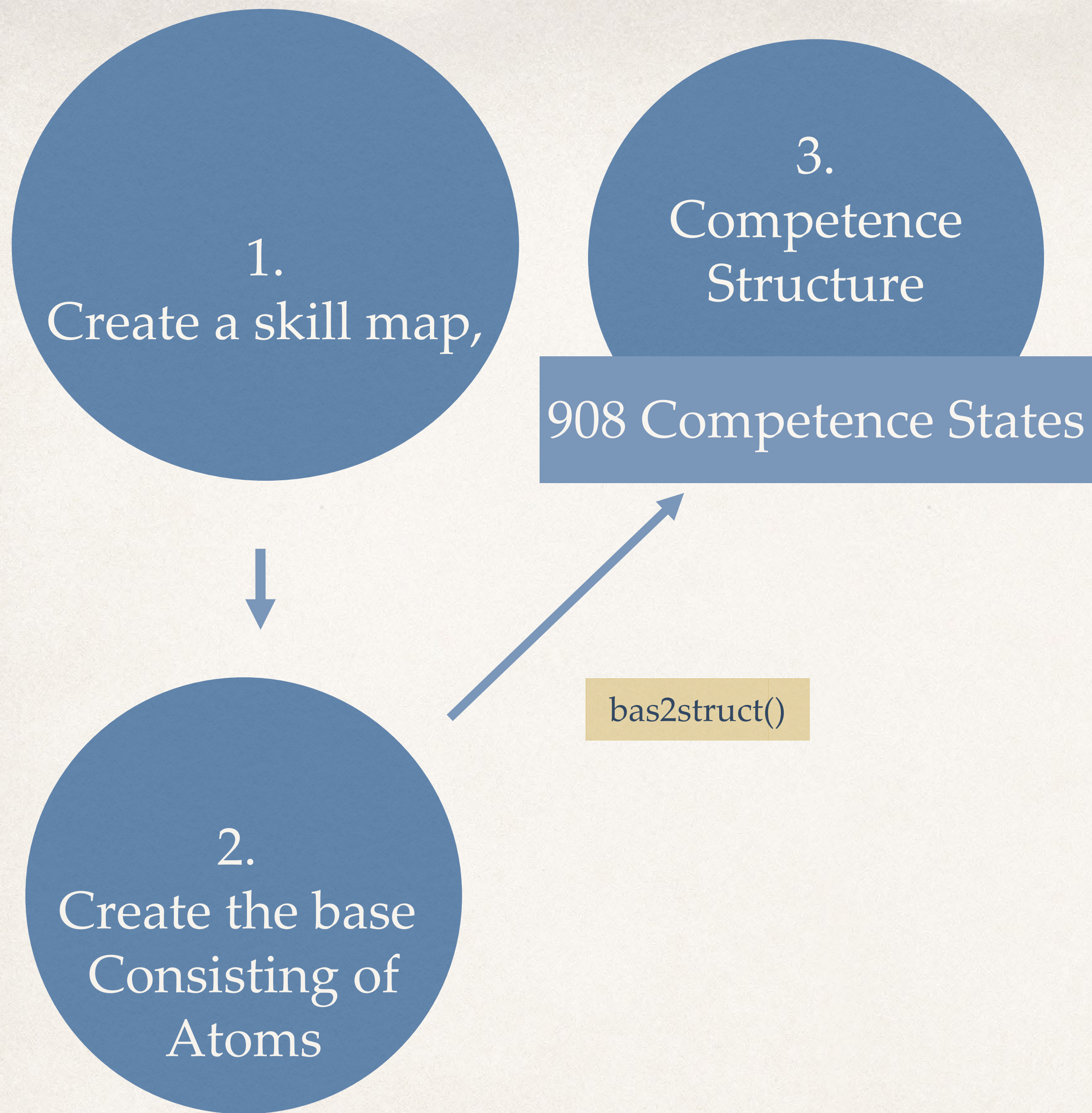
1.
Create a skill map,



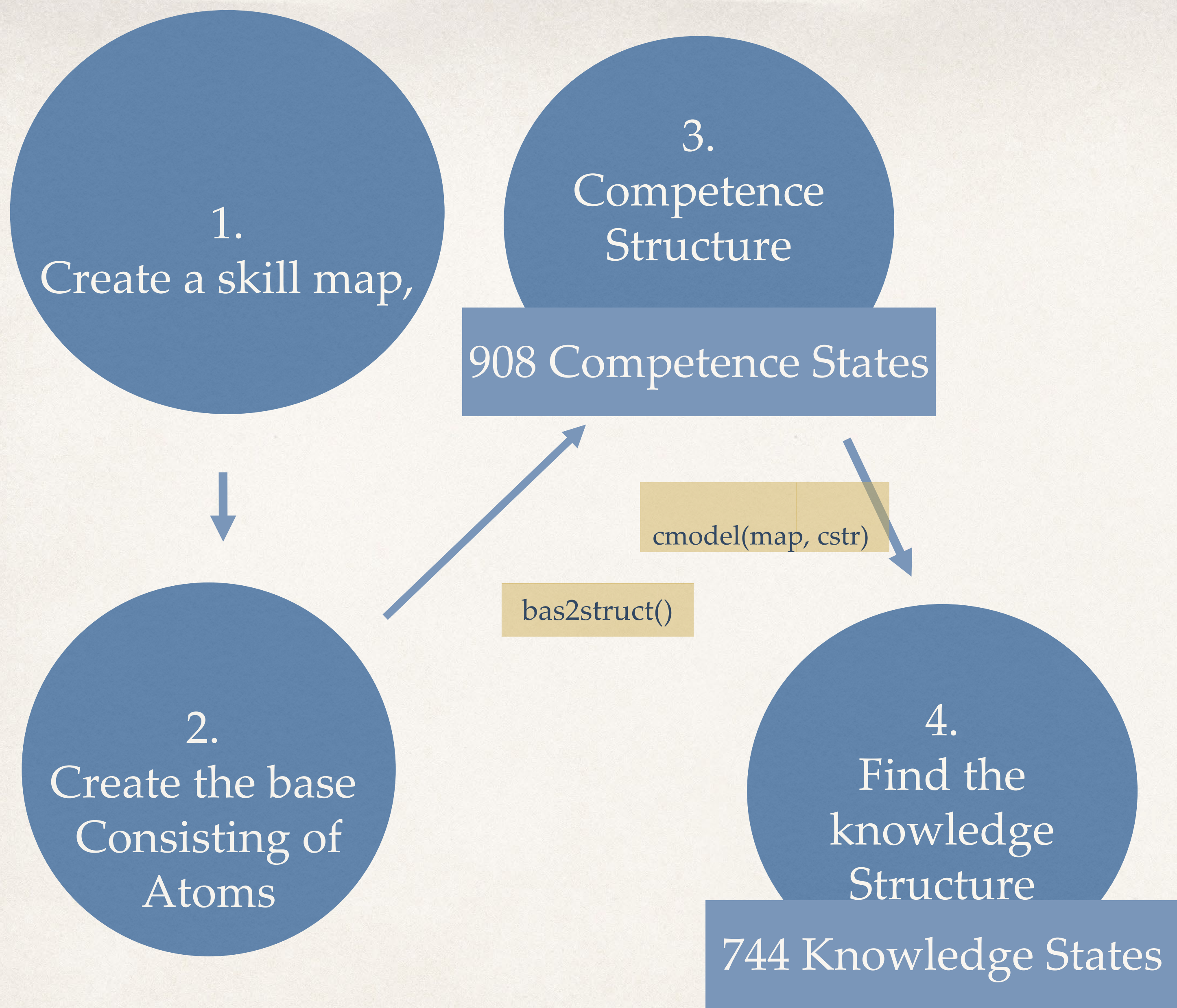
2.
Create the base
Consisting of
Atoms



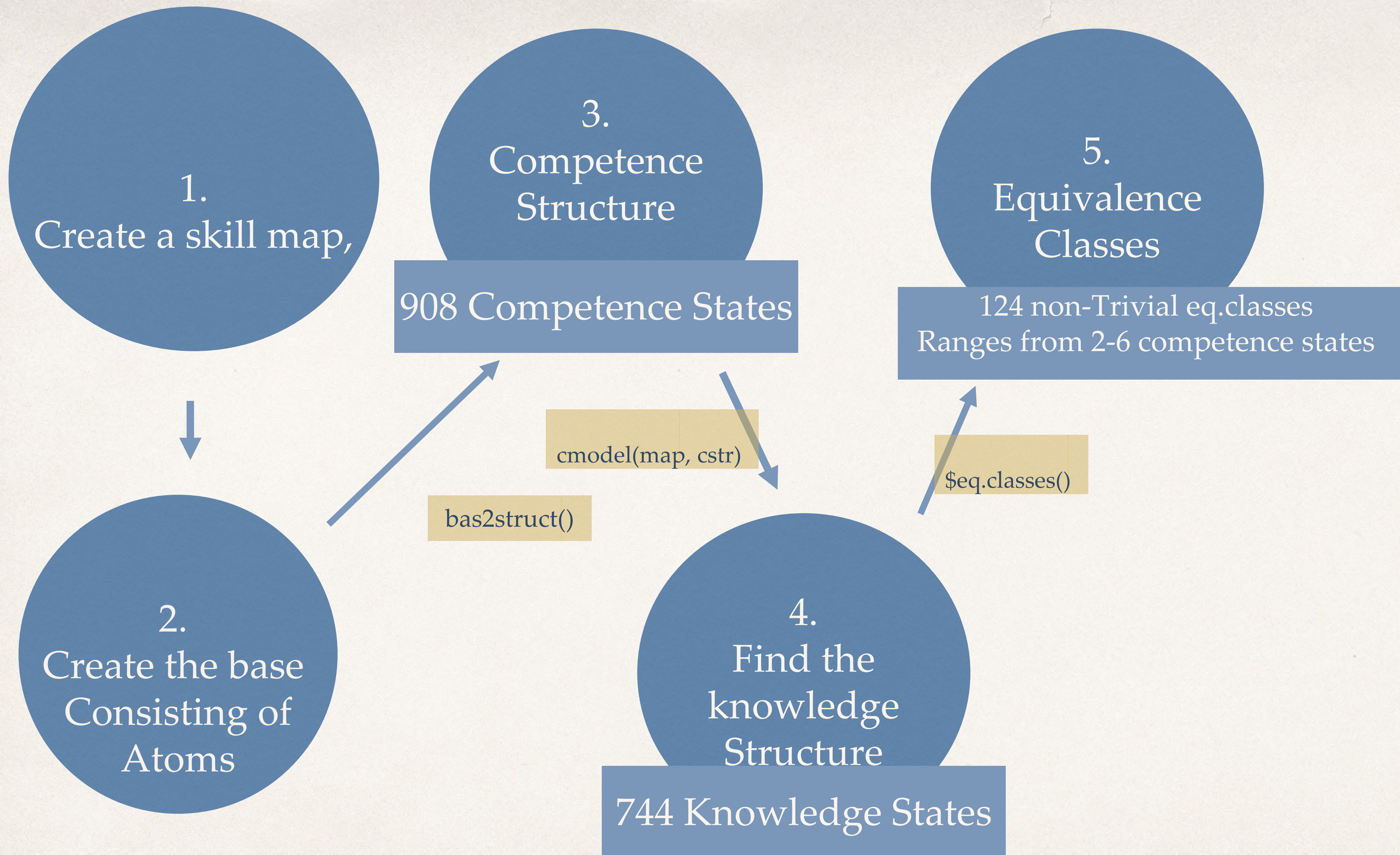
The Procedure (3)



The Procedure (3)



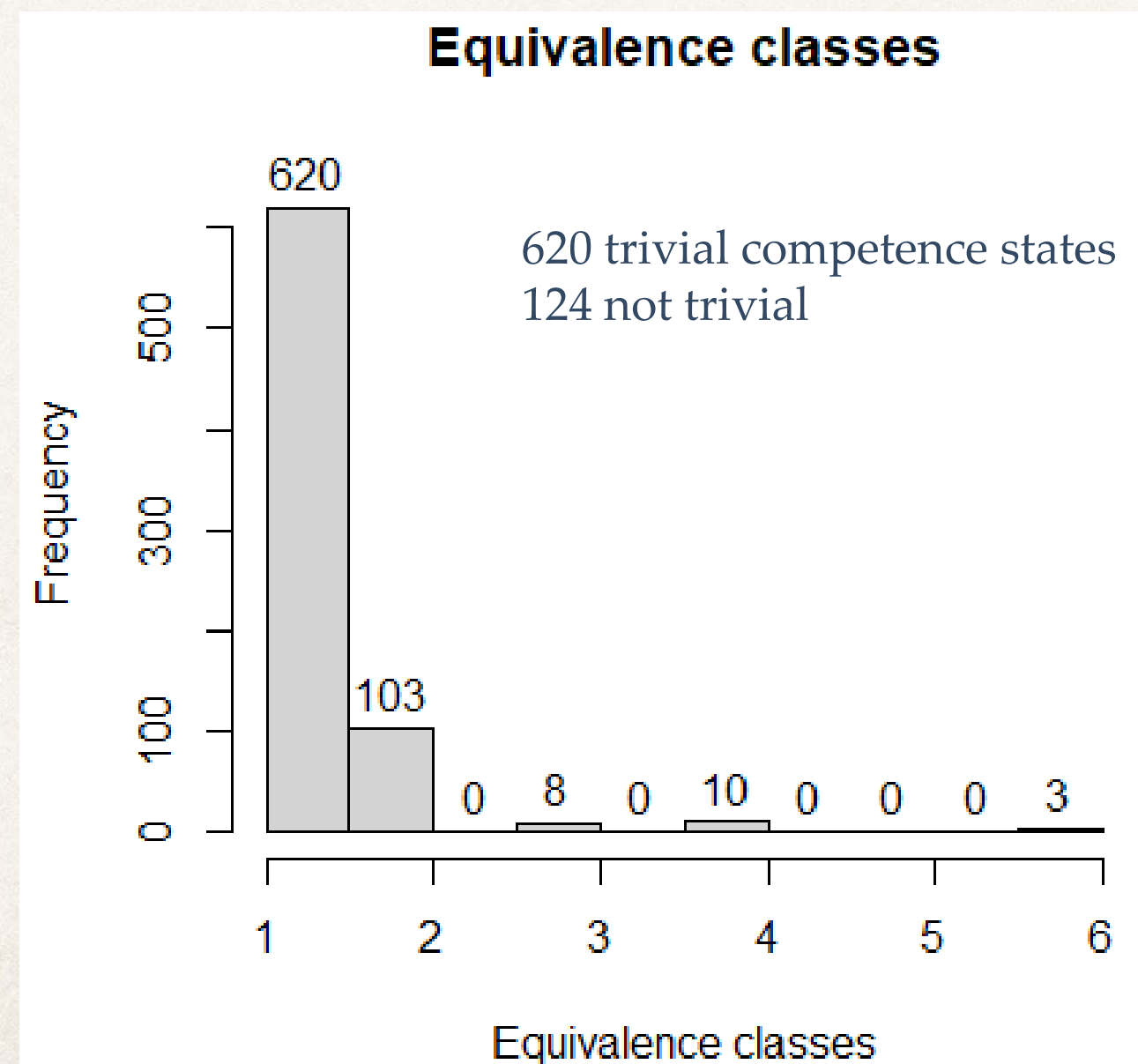
The Procedure (3)



The Procedure (3)

EQUIVALENCE CLASSES

When different **competence states** delineate the same **knowledge state**.



EX:
5 students have
5 different set of **abilities**
(**competence states**)..
..but they all result in
solving the same **exercise**
(**knowledge state**)

So, we cannot know their real
abilities!

1.
Create a skill map,
create the base

2.
Define the base
Consisting of
Atoms

3.
Competence
Structure

908 Competence States

5.
Equivalence
Classes

124 non-Trivial eq.classes
Ranges from 2-6 competence states

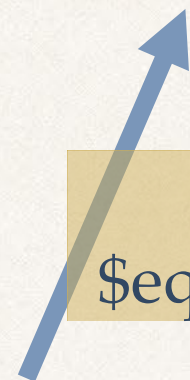
4.
Find the
knowledge
Structure

744 Knowledge States



bas2struct()

cmodel(map, cstr)



\$eq.classes()



1.
Create a skill map,
create the base

2.
Define the base
Consisting of
Atoms

3.
Competence
Structure
908 Competence States

5.
Equivalence
Classes
124 non-Trivial eq.classes
Ranges from 2-6

4.
Find the
knowledge
Structure
744 Knowledge States

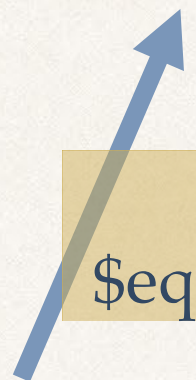
6.
Shorten the
Test



cmodel(map, cstr)



\$eq.classes()



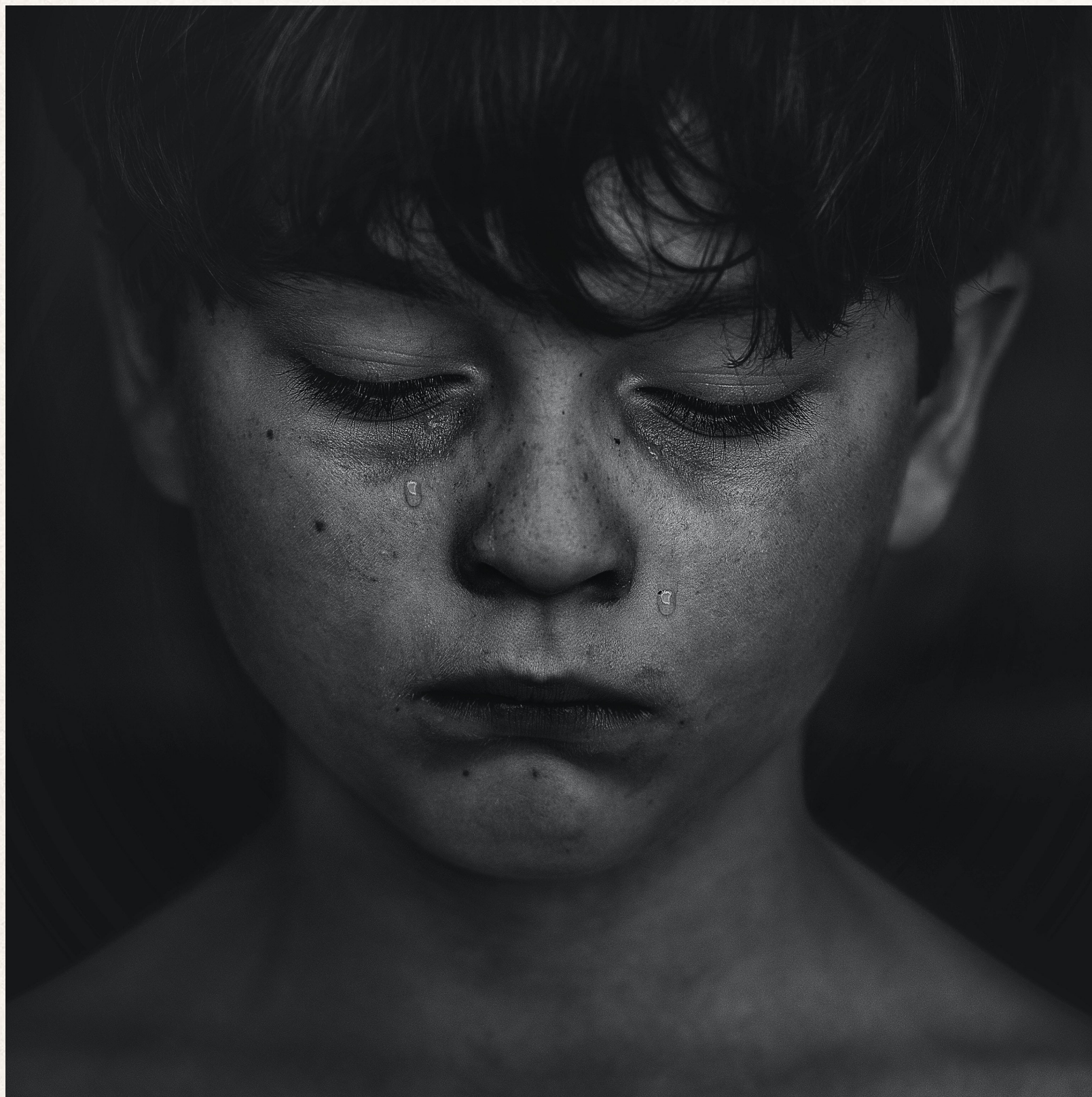
bas2struct()

cdp(cstr, design, comp)

Conclusion

- ❖ We shortened the test **from 29 to 17 items**
- ❖ Examining **17 Competencies**, as **informative as the original test**
- ❖ From deterministic point of view the test is as **good as the original one**.
In practice smaller tests are less reliable than longer ones.
- ❖ Because when careless errors and lucky guesses occur, shorter tests are less reliable than longer ones, meaning that it is more difficult to recover the true competence state of the individual from his/her item responses

Why Improve Tests?



Main References

Anselmi, P., Heller, J., Stefanutti, L., & Robusto, E. (2022). Constructing, improving, and shortening tests for skill assessment. *Journal of Mathematical Psychology*, 106, 102621.

Heller, J., Stefanutti, L., Anselmi, P., & Robusto, E. (2015). On the link between cognitive diagnostic models and knowledge space theory. *Psychometrika*, 80(4), 995-1019.

Spoto, A., Stefanutti, L., & Vidotto, G. (2010). Knowledge space theory, formal concept analysis, and computerized psychological assessment. *Behavior Research Methods*, 42, 342-350.

Su, Y. L., Choi, K. M., Lee, W. C., Choi, T., & McAninch, M. (2013). Hierarchical cognitive diagnostic analysis for TIMSS 2003 mathematics. *Centre for Advanced Studies in Measurement and Assessment*, 35, 1-71.

R Core Team (2013). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL <http://www.R-project.org/>.

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Special thanks to Prof. Pasquale Anselmi!



Table 4. Q-Matrix of Booklet 1 for the Eighth Grade TIMSS 2003 Mathematics Test

Item\Attribute	1	2	3	4	5	6	7	8	9	11	12	13	14	15	Sum
1 M012001	1	0	0	0	0	0	0	0	0	0	0	1	0	1	3
2 M012002	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
3 M012004	0	1	0	0	0	0	1	0	0	0	0	0	0	0	2
4 M012040	0	0	0	0	0	1	1	0	0	0	0	0	0	0	2
5 M012041	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6 M012042	0	0	0	0	1	0	0	0	1	0	0	0	0	0	2
7 M032570	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
8 M032643	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
9 M012016	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
10 M012017	0	0	0	0	0	1	0	0	0	1	0	0	1	0	3
11 M022251	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
12 M022185	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
13 M022191	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
14 M022194	0	1	0	0	0	0	0	0	0	0	0	0	1	0	2
15 M022196	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
16 M022198	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
17 M022199	0	0	1	0	0	0	0	0	1	0	0	1	0	1	4
18 M022043	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
19 M022046	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
20 M022050	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
21 M022057	0	1	0	0	0	0	0	1	0	0	0	0	0	0	2
22 M022066	0	0	1	0	0	0	0	0	0	0	0	0	0	1	2
23 M022232	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
24 M022234B	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25 M032142	0	1	0	0	0	0	1	0	0	0	0	0	0	0	2
26 M032198	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
27 M032640	0	0	0	0	0	0	0	0	0	1	0	0	1	0	3
28 M032755	0	1	0	0	0	0	1	1	0	1	0	0	0	0	4
29 M032163	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
Sum	3	7	3	3	5	6	4	2	4	3	1	3	3	3	

Table 5. Q-Matrix of Booklet 2 for the Eighth Grade TIMSS 2003 Mathematics Test

Item\Attribute	1	2	3	4	5	6	7	8	9	10	11	13	14	15	Sum
1 M012016	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
2 M012017	0	0	0	0	0	1	0	0	0	0	1	0	1	0	3
3 M022251	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
4 M022185	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
5 M022191	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
6 M022194	0	1	0	0	0	0	0	0	0	0	0	0	1	0	2
7 M022196	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
8 M022198	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
9 M022199	0	0	1	0	0	0	0	0	1	0	0	1	0	1	4
10 M012025	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
11 M012027	1	1	0	0	0	0	0	1	0	0	0	0	0	0	3
12 M012029	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
13 M022139	1	1	0	0	0	0	0	1	0	0	0	0	0	0	3
14 M022144	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
15 M022253	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2
16 M022156	0	1	0	0	0	1	0	0	0	0	0	0	0	0	2
17 M022104	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
18 M022106	1	0	0	0	0	1	1	0	0	0	0	0	0	0	3
19 M022110	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
20 M032307	0	0	0	0	0	0	0	0	1	1	0	0	0	1	3
21 M032523	0	0	0	0	0	0	0	0	0	1	0	0	0	1	2
22 M032701	0	1	0	0	0	1	0	0	0	0	0	0	0	0	2
23 M032704	1	1	0	0	0	0	1	0	0	0	0	0	0	0	3
24 M032525	0	0	0	0	0	1	0	0	1	0	0	0	0	0	2
25 M032381	0	0	1	0	0	0	1	0	0	0	0	0	0	0	2
26 M032416	0	0	1	0	0	0	0	0	0	0	0	0	0	1	2
27 M032160	0	1	0	0	0	0	1	0	0	0	0	0	0	0	2
28 M032540	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
29 M032698	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2
30 M032529	0	1	0	0	0	0	0	1	0	0	0	0	0	0	2
Sum	4	8	4	3	5	9	5	3	4	2	1	1	4	4	

If you feel inspired improve the test further, by incorporating elements from booklet 2 - don't hesitate to slack us your findings ;)